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Projectile problems are presented along with detailed solutions. These problems may be better understood when projectile equations are first reviewed. An interactive html 5 applet may be used to better understand the projectile equations. Problems with Detailed Solutions Problem 1 An object is launched at a velocity of 20 m/s in a direction making an angle of 25 upward with the horizontal. a) What is the maximum height reached by the object? b) What is the total flight time (between launch and touching the ground) of the object? c) What is the horizontal range (maximum x above ground) of the object? d) What is the magnitude of the velocity of the object just before it hits the ground? Solution to Problem 1 Problem 2 A projectile is launched from point O at an angle of 22 with an initial velocity of 15 m/s up an incline plane that makes an angle of 10 with the horizontal. The projectile hits the incline plane at point M. a) Find the time it takes for the projectile to hit the incline plane. b)Find the distance OM. Solution to Problem 2 Problem 3 A projectile is to be launched at an angle of 30 so that it falls beyond the pond of length 20 meters as shown in the figure. a) What is the range of values of the initial velocity so that the projectile falls between points M and N? Solution to Problem 3 Problem 4 A ball is kicked at an angle of 35 with the ground. a) What should be the initial velocity of the ball so that it hits a target that is 30 meters away at a height of 1.8 meters? b) What is the time for the ball to reach the target? Solution to Problem 4 Problem 5 A ball kicked from ground level at an initial velocity of 60 m/s and an angle with ground reaches a horizontal distance of 200 meters. a) What is the size of angle ? b) What is time of flight of the ball? Solution to Problem 5 Problem 6 A ball of 600 grams is kicked at an angle of 35 with the ground with an initial velocity V0. a) What is the initial velocity V0 of the ball if its kinetic energy is 22 Joules when its height is maximum? b) What is the maximum height reached by the ball Solution to Problem 6 Problem 7 A projectile starting from ground hits a target on the ground located at a distance of 1000 meters after 40 seconds. a) What is the size of the angle ? b) At what initial velocity was the projectile launched? Solution to Problem 7 Problem 8 The trajectory of a projectile launched from ground is given by the equation y = -0.025 x2 + 0.5 x, where x and y are the coordinate of the projectile on a rectangular system of axes. a) Find the initial velocity and the angle at which the projectile is launched. Solution to Problem 8 Problem 9 Two balls A and B of masses 100 grams and 300 grams respectively are pushed horizontally from a table of height 3 meters. Ball has is pushed so that its initial velocity is 10 m/s and ball B is pushed so that its initial velocity is 15 m/s. a) Find the time it takes each ball to hit the ground. b) What is the difference in the distance between the points of impact of the two balls on the ground? Solution to Problem 9 More References and Links Share copy and redistribute the material in any medium or format for any purpose, even commercially. Adapt remix, transform, and build upon the material for any purpose, even commercially. The licensor cannot revoke these freedoms as long as you follow the license terms. Attribution You must give appropriate credit , provide a link to the license, and indicate if changes were made . You may do so in any reasonable manner, but not in any way that suggests the licensor endorses you or your use. ShareAlike If you remix, transform, or build upon the material, you must distribute your contributions under the same license as the original. No additional restrictions You may not apply legal terms or technological measures that legally restrict others from doing anything the license permits. You do not have to comply with the license for elements of the material in the public domain or where your use is permitted by an applicable exception or limitation . No warranties are given. The license may not give you all of the permissions necessary for your intended use. For example, other rights such as publicity, privacy, or moral rights may limit how you use the material. 0 ratings0% found this document useful (0 votes)9K views1. A kicked football leaves the ground at an angle of 60 with an initial speed of 10 m/s. The maximum height reached is calculated to be 3.75 meters. 2. A body is projected upward at an anAI-enhanced title and descriptionSaveSave Projectile Motion For Later0%0% found this document useful, undefined Projectile problems are presented along with detailed solutions. These problems may be better understood when projectile equations are first reviewed. An interactive html 5 applet may be used to better understand the projectile equations. Problems with Detailed Solutions Problem 1 An object is launched at a velocity of 20 m/s in a direction making an angle of 25 upward with the horizontal. a) What is the maximum height reached by the object? b) What is the total flight time (between launch and touching the ground) of the object? c) What is the horizontal range (maximum x above ground) of the object? d) What is the magnitude of the velocity of the object just before it hits the ground? Solution to Problem 1 Problem 2 A projectile is launched from point O at an angle of 22 with an initial velocity of 15 m/s up an incline plane that makes an angle of 10 with the horizontal. The projectile hits the incline plane at point M. a) Find the time it takes for the projectile to hit the incline plane. b)Find the distance OM. 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Unsourced material may be challenged and removed.Find sources:"Projectile motion"news newspapers books scholar JSTOR (November 2019) (Learn how and when to remove this message) (Learn how and when to remove this message)Parabolic trajectories of water jetsComponents of initial velocity of parabolic throwingBallistic trajectories are parabolic if gravity is homogeneous and elliptic if it is radial.In physics, projectile motion describes the motion of an object that is launched into the air and moves under the influence of gravity alone, with air resistance neglected. In this idealized model, the object follows a parabolic path determined by its initial velocity and the constant acceleration due to gravity. The motion can be decomposed into horizontal and vertical components: the horizontal motion occurs at a constant velocity, while the vertical motion experiences uniform acceleration.This framework, which lies at the heart of classical mechanics, is fundamental to a wide range of applicationsfrom engineering and ballistics to sports science and natural phenomena.Galileo Galilei showed that the trajectory of a given projectile is parabolic, but the path may also be straight in the special case when the object is thrown directly upward or downward. The study of such motions is called ballistics, and such a trajectory is described as ballistic. The only force of mathematical significance that is actively exerted on the object is gravity, which acts downward, thus imparting to the object a downward acceleration towards Earth's center of mass. Due to the object's inertia, no external force is needed to maintain the horizontal velocity component of the object's motion.Taking other forces into account, such as aerodynamic drag or internal propulsion (such as in a rocket), requires additional analysis. A ballistic missile is a missile only guided during the relatively brief initial powered phase of flight, and whose remaining course is governed by the laws of classical mechanics.Ballistics (from Ancient Greek bllein'to throw') is the science of dynamics that deals with the flight, behavior and effects of projectiles, especially bullets, unguided bombs, rockets, or the like; the science or art of designing and accelerating projectiles so as to achieve a desired performance.Trajectories of a projectile with air drag and varying initial velocitiesThe elementary equations of ballistics neglect nearly every factor except for initial velocity, the launch angle and a gravitational acceleration assumed constant. Practical solutions of a ballistics problem often require considerations of air resistance, cross winds, target motion, acceleration due to gravity varying with height, and in such problems as launching a rocket from one point on the Earth to another, the horizon's distance vs curvature R of the Earth (its local speed of rotation v (l a t) = R (l a t) {\textstyle v(lat)=\omega R(lat)}). Detailed mathematical solutions of practical problems typically do not have closed-form solutions, and therefore require numerical methods to address.In projectile motion, the horizontal motion and the vertical motion are independent of each other; that is, neither motion affects the other. This is the principle of compound motion established by Galileo in 1638,[1] and used by him to prove the parabolic form of projectile motion.[2]The horizontal and vertical components of a projectile's velocity are independent of each other.A ballistic trajectory is a parabola with homogeneous acceleration, such as in a space ship with constant acceleration in absence of other forces. On Earth the acceleration changes magnitude with altitude as g (y) = g 0 / (1 + y / R) 2 {\textstyle g(y)=g_{0}/(1+y/R)^{2}} and direction (faraway targets) with latitude/longitude along the trajectory. This causes an elliptic trajectory, which is very close to a parabola on a small scale. However, if an object was thrown and the Earth was suddenly replaced with a black hole of equal mass, it would become obvious that the ballistic trajectory is part of an elliptic orbit around that "black hole", and not a parabola that extends to infinity. At higher speeds the trajectory can also be circular (cosmonautics at LEO?, geostationary satellites at 5 5 6 {\textstyle {\frac {5}{6}}} R), parabolic or hyperbolic (unless distorted by other objects like the Moon or the Sun).In this article a homogeneous gravitational acceleration (g = g 0) {\textstyle (g=g_{0})} is assumed.Since there is acceleration only in the vertical direction, the velocity in the horizontal direction is constant, being equal to v 0 cos {\displaystyle \mathbf {v} _{0}\cos \theta } . The vertical motion of the projectile is the motion of a particle during its free fall. Here the acceleration is constant, being equal to g.[note 1] The components of the acceleration are: a x = 0 {\displaystyle a_{x}=0} , a y = g {\displaystyle a_{y}=-g} .**The y acceleration can also be referred to as the force of the earth (F g / m) {\textstyle (-F_{g}/m)} on the object(s) of interest.Let the projectile be launched with an initial velocity v (0) v 0 {\displaystyle \mathbf {v} (0)\equiv \mathbf {v} _{0}} , which can be expressed as the sum of horizontal and vertical components as follows: v 0 = v 0 x x ^ + v 0 y y ^ {\displaystyle \mathbf {v} _{0}=v_{0x}\mathbf {\hat {x}} +v_{0y}\mathbf {\hat {y}} } .The components v 0 x {\displaystyle v_{0x}} and v 0 y {\displaystyle v_{0y}} can be found if the initial launch angle is known: v 0 x = v 0 cos ( ) {\displaystyle v_{0x}=v_{0}\cos(\theta)} , v 0 y = v 0 sin ( ) {\displaystyle v_{0y}=v_{0}\sin(\theta)} The horizontal component of the velocity of the object remains unchanged throughout the motion. The vertical component of the velocity changes linearly,[note 2] because the acceleration due to gravity is constant. The accelerations in the x and y directions can be integrated to solve for the components of velocity at any time t, as follows: v x = v 0 cos ( ) {\displaystyle v_{x}=v_{0}\cos(\theta)} , v y = v 0 sin ( ) g t {\displaystyle v_{y}=v_{0}\sin(\theta)-gt} .The magnitude of the velocity (under the Pythagorean theorem, also known as the triangle law): v = v x 2 + v y 2 {\displaystyle v={\sqrt {v_{x}^{2}+v_{y}^{2}}}} .Displacement and coordinates of parabolic throwingAt any time t {\displaystyle t} , the projectile's horizontal and vertical displacement are: x = v 0 t cos ( ) {\displaystyle x=v_{0}t\cos(\theta)} , y = v 0 t sin ( ) 1 2 g t 2 {\displaystyle y=v_{0}t\sin(\theta)-{\frac {1}{2}}gt^{2}} .The magnitude of the displacement is: r = x 2 + y 2 {\displaystyle \Delta r={\sqrt {x^{2}+y^{2}}}} .Consider the equations, x = v 0 t cos ( ) {\displaystyle x=v_{0}t\cos(\theta)} and y = v 0 t sin ( ) 1 2 g t 2 {\displaystyle y=v_{0}t\sin(\theta)-{\frac {1}{2}}gt^{2}} .[3]If t is eliminated between these two equations the following equation is obtained: y = tan ( ) x g 2 v 0 2 cos 2 x 2 = tan x (1 x R) . {\displaystyle y=\tan(\theta)\cdot x-{\frac {g} {2v_{0}^{2}\cos ^{2}\theta }}\cdot x^{2}=\tan \theta \cdot x\left(1-{\frac {x}{R}}\right).} Here R is the range of a projectile.Since g, , and v0 are constants, the above equation is of the form y = a x + b x 2 {\displaystyle y=ax+bx^{2}} ,in which a and b are constants. This is the equation of a parabola, so the path is parabolic. The axis of the parabola is vertical.If the projectile's position (x,y) and launch angle (or) are known, the initial velocity can be found solving for v0 in the afore-mentioned parabolic equation: v 0 = x 2 g x sin 2 2 y cos 2 {\displaystyle v_{0}={\sqrt {{x^{2}g} \over {x\sin 2\theta -2y\cos ^{2}\theta }}}} .The parabolic trajectory of a projectile can also be expressed in polar coordinates instead of Cartesian coordinates. In this case, the position has the general formula r ( ) = 2 v 0 2 cos 2 | g | (tan sec tan sec) {\displaystyle r(\phi)={\frac {2v_{0}^{2}\cos ^{2}\theta }{|g|}}\left(\tan \theta \sec \phi -\tan \phi \sec \phi \right)} .In this equation, the origin is the midpoint of the horizontal range of the projectile, and if the ground is flat, the parabolic arc is plotted in the range 0 {\displaystyle 0\leq \phi \leq \pi } . This expression can be obtained by transforming the Cartesian equation as stated above by y = r sin {\displaystyle y=r\sin \phi } and x = r cos {\displaystyle x=r\cos \phi } .The total time t for which the projectile remains in the air is called the time-of-flight. y = v 0 t sin ( ) 1 2 g t 2 {\displaystyle y=v_{0}t\sin(\theta)-{\frac {1}{2}}gt^{2}} After the flight, the projectile returns to the horizontal axis (x-axis), so y = 0 {\displaystyle y=0} . v 0 t sin ( ) = 1 2 g t 2 {\displaystyle v_{0}t\sin(\theta)={\frac {1}{2}}gt^{2}} v 0 sin ( ) = 1 2 g t {\displaystyle v_{0}\sin(\theta)={\frac {1}{2}}gt} t = 2 v 0 sin ( ) g {\displaystyle t={\frac {2v_{0}\sin(\theta)}{g}}} Note that we have neglected air resistance on the projectile.If the starting point is at height y0 with respect to the point of impact, the time of flight is: t = d v cos = v sin + (v sin) 2 + 2 g y 0 g {\displaystyle t={\frac {d}{v\cos \theta }}={\frac {v\sin \theta +{\sqrt {(v\sin \theta)^{2}+2gy_{0}}}}{g}}} As above, this expression can be reduced (y0 is 0) to t = v sin + (v sin) 2 g = 2 v sin g , {\displaystyle t={\frac {v\sin {\theta }+{\sqrt {(v\sin {\theta })^{2}}}}{g}}={\frac {2v\sin {\theta }}{g}},} = 2 v sin (45) | g | = 2 v | g | {\displaystyle {\frac {2v\sin {(45^{\circ })}}{|g|}}={\frac {{\sqrt {2}}v}{|g|}}} if equals 45.As shown above in the Displacement section, the horizontal and vertical velocity of a projectile are independent of each other.Because of this, we can find the time to reach a target using the displacement formula for the horizontal velocity: x = v 0 t cos ( ) {\displaystyle x=v_{0}t\cos(\theta)} x t = v 0 cos ( ) {\displaystyle {\frac {x} {t}}=v_{0}\cos(\theta)} t = x v 0 cos ( ) {\displaystyle t={\frac {x}{v_{0}\cos(\theta)}}} This equation will give the total time t the projectile must travel for to reach the target's horizontal displacement, neglecting air resistance.Maximum height of projectileThe greatest height that the object will reach is known as the peak of the object's motion.The increase in height will last until v y = 0 {\displaystyle v_{y}=0} , that is, 0 = v 0 sin ( ) g t h {\displaystyle 0=v_{0}\sin(\theta)-gt_{h}} .Time to reach the maximum height(h): t h = v 0 sin ( ) | g | {\textstyle t_{h}={\frac {v_{0}\sin(\theta)}{|g|}}} .For the vertical displacement of the maximum height of the projectile: h = v 0 t h sin ( ) 1 2 g t h 2 {\displaystyle h=v_{0}t_{h}\sin(\theta)-{\frac {1}{2}}gt_{h}^{2}} h = v 0 2 sin 2 ( ) 2 | g | {\displaystyle h={\frac {v_{0}^{2}\sin ^{2}(\theta)}{2|g|}}} The maximum reachable height is obtained for =90: h m a x = v 0 2 2 | g | {\displaystyle h_{\mathrm {max} }={\frac {v_{0}^{2}}{2|g|}}} If the projectile's position (x,y) and launch angle () are known, the maximum height can be found by solving for h in the following equation: h = (x tan) 2 4 (x tan y) . {\displaystyle h={\frac {(x\tan \theta)^{2}}{4(x\tan \theta -y)}}.} Angle of elevation () at the maximum height is given by: = arctan tan 2 {\displaystyle \phi =\arctan {\tan \theta \over 2}} The relation between the range d on the horizontal plane and the maximum height h reached at t d 2 {\displaystyle {\frac {t_{d}}{2}}} is: h = d tan 4 {\displaystyle h={\frac {d\tan \theta }{4}}} Proof h = v 0 2 sin 2 2 | g | {\displaystyle h={\frac {v_{0}^{2}\sin ^{2}\theta }{2|g|}}} d = v 0 2 sin 2 | g | {\displaystyle d={\frac {v_{0}^{2}\sin 2\theta }{|g|}}} h d = v 0 2 sin 2 2 | g | {\displaystyle {\frac {h}{d}}={\frac {v_{0}^{2}\sin ^{2}\theta }{2|g|}}} g v 0 2 sin 2 {\displaystyle {\frac {g}{v_{0}^{2}\sin 2\theta }}} h d = sin 2 4 sin cos {\displaystyle {\frac {h}{d}}={\frac {\sin ^{2}\theta }{4\sin \theta \cos \theta }}} h = d tan 4 {\displaystyle h={\frac {d\tan \theta }{4}}} .If h = R {\displaystyle h=R} = arctan (4) 76.0 {\displaystyle \theta =\arctan(4)\approx 76.0^{\circ }} Main article: Range of a projectileThe maximum distance of projectileThe range and the maximum height of the projectile do not depend upon its mass. Hence range and maximum height are equal for all bodies that are thrown with the same velocity and direction. The horizontal range d of the projectile is the horizontal distance it has traveled when it returns to its initial height (y = 0 {\textstyle y=0}). 0 = v 0 t d sin ( ) 1 2 g t d 2 {\displaystyle 0=v_{0}t_{d}\sin(\theta)-{\frac {1}{2}}gt_{d}^{2}} .Time to reach ground: t d = 2 v 0 sin ( ) | g | {\displaystyle t_{d}={\frac {2v_{0}\sin(\theta)}{|g|}}} .From the horizontal displacement the maximum distance of the projectile: d = v 0 t d cos ( ) {\displaystyle d=v_{0}t_{d}\cos(\theta)} ,so[note 3] d = v 0 2 | g | sin (2) . {\displaystyle d={\frac {v_{0}^{2}}{|g|}}\sin(2\theta).} Note that d has its maximum value when sin (2) = 1 , {\displaystyle \sin(2\theta)=1,} which necessarily corresponds to 2 = 90 {\textstyle 2\theta =90^{\circ }} , or = 45 {\textstyle \theta =45^{\circ }} .Trajectories of projectiles launched at different elevation angles but the same speed of 10 m/s, in a vacuum and uniform downward gravity field of 10 m/s2. Points are at 0.05 s intervals and length of their tails is linearly proportional to their speed. t = time from launch, T = time of flight, R = range and H = highest point of trajectory (indicated with arrows).The total horizontal distance (d) traveled. d = v cos | g | (v sin + (v sin) 2 + 2 g y 0) {\displaystyle d={\frac {v\cos \theta }{|g|}}\left(v\sin \theta +{\sqrt {(v\sin \theta)^{2}+2gy_{0}}}\right)} When the surface is flat (initial height of the object is zero), the distance traveled:[4] d = v 2 sin (2) | g | {\displaystyle d={\frac {v^{2}\sin(2\theta)}{|g|}}} Thus the maximum distance is obtained if is 45 degrees. This distance is: d m a x = v 2 | g | {\displaystyle d_{\mathrm {max} }={\frac {v^{2}}{|g|}}} According to the work-energy theorem the vertical component of velocity is: v y 2 = (v 0 sin) 2 2 g y {\displaystyle v_{y}^{2}=(v_{0}\sin \theta)^{2}-2gy} .These formulae ignore aerodynamic drag and also assume that the landing area is at uniform height 0.The "angle of reach" is the angle () at which a projectile must be launched in order to go a distance d, given the initial velocity v. sin (2) = g d v 2 {\displaystyle \sin(2\theta)={\frac {gd}{v^{2}}}} There are two solutions: = 1 2 arcsin (g d v 2) {\displaystyle \theta ={\frac {1}{2}}\arcsin \left({\frac {gd}{v^{2}}}\right)} (shallow trajectory)and because sin (2) = cos (2 90) {\displaystyle \sin(2\theta)=\cos(2\theta -90^{\circ })} , = 45 + 1 2 arccos (g d v 2) {\displaystyle \theta =45^{\circ }+{\frac {1}{2}}\arccos \left({\frac {gd}{v^{2}}}\right)} (steep trajectory)Vacuum trajectory of a projectile for different launch angles. Launch speed is the same for all angles, 50m/s, and "g" is 10m/s2.To hit a target at range x and altitude y when fired from (0,0) and with initial speed v, the required angle(s) of launch are: = arctan (v 2 v 4 g (g x 2 + 2 y v 2) g x) {\displaystyle \theta =\arctan {\left({\frac {v^{2}\pm {\sqrt {v^{4}-g(gx^{2}+2yv^{2})}}}{gx}}\right)}} The two roots of the equation correspond to the two possible launch angles, so long as they aren't imaginary, in which case the initial speed is not great enough to reach the point (x,y) selected. This formula allows one to find the angle of launch needed without the restriction of y = 0 {\textstyle y=0} .One can also ask what launch angle allows the lowest possible launch velocity. This occurs when the two solutions above are equal, implying that the quantity under the square root sign is zero. This, tan = v2/gx, requires solving a quadratic equation for v 2 {\displaystyle v^{2}} ,[5] and we find v 2 / g = (2 g y g 4 y 2 + 4 (1 x) 2) / 2 g = y y 2 + x 2 . {\displaystyle v^{2}/g=(2gy\pm g{\sqrt {4y^{2}+4(1x)^{2}}})/2g=y\pm {\sqrt {y^{2}+x^{2}}}.} This gives = arctan (y / x + y 2 / x 2 + 1) . {\displaystyle \theta =\arctan \left(y/x+{\sqrt {y^{2}/x^{2}+1}}\right).} If we denote the angle whose tangent is y/x by ,[6] then tan = sin + 1 cos , {\displaystyle \tan \theta ={\frac {\sin \alpha +1}{\cos \alpha }},} its reciprocal: tan (/ 2) = cos sin + 1 , {\displaystyle \tan(\pi /2-\theta)={\frac {\cos \alpha }{\sin \alpha +1}},} [7] cos 2 (/ 2) = 1 2 (sin + 1) {\displaystyle \cos ^{2}(\pi /2-\theta)={\frac {1}{2}}(\sin \alpha +1)} 2 cos 2 (/ 2) 1 = cos (/ 2) {\displaystyle 2\cos ^{2}(\pi /2-\theta)-1=\cos(\pi /2-\alpha)} This implies = / 2 1 2 (/ 2) . {\displaystyle \theta =\pi /2-{\frac {1}{2}}(\pi /2-\alpha).} In other words, the launch should be at the angle (/ 2 +) / 2 {\textstyle (\pi /2+\alpha)/2} halfway between the target and zenith (vector opposite to gravity).The length of the parabolic arc traced by a projectile, L, given that the height of launch and landing is the same (there is no air resistance), is given by the formula: L = v 0 2 2 g (2 sin + cos 2 ln 1 + sin 1 sin) = v 0 2 g (sin + cos 2 tanh 1 (sin)) {\displaystyle L={\frac {v_{0}^{2}}{2g}}\left(2\sin \theta +\cos ^{2}\theta \cdot \ln {\frac {1+\sin \theta }{1-\sin \theta }}\right)={\frac {v_{0}^{2}}{g}}\left(\sin \theta +\cos ^{2}\theta \cdot \tanh ^{-1} (\sin \theta)\right)} where v 0 {\displaystyle v_{0}} is the initial velocity, {\displaystyle \theta } is the launch angle and g {\displaystyle g} is the acceleration due to gravity as a positive value. The expression can be obtained by evaluating the arc length integral for the height-distance parabola between the bounds initial and final displacement (i.e. between 0 and the horizontal range of the projectile) such that: L = 0 r a n g e 1 + (d y d x) 2 d x = 0 v 0 2 sin (2) / g 1 + (tan g v 0 2 cos 2 x) 2 d x . {\displaystyle L=\int _{0}^{\mathrm {range} }{\sqrt {1+\left({\frac {\mathrm {d} y}{\mathrm {d} x}}\right)^{2}}}\,\mathrm {d} x=\int _{0}^{v_{0}^{2}\sin(2\theta)/g}{\sqrt {1+\left(\tan \theta -{g \over {v_{0}^{2}\cos ^{2}\theta }}x\right)^{2}}}\,\mathrm {d} x.} If the time-of-flight is t, L = 0 t v x 2 + v y 2 d t = 0 2 v 0 sin / g (g t) 2 2 g v 0 sin t + v 0 2 d t . {\displaystyle L=\int _{0}^{t}{\sqrt {v_{x}^{2}+v_{y}^{2}}}\,\mathrm {d} t=\int _{0}^{2v_{0}\sin \theta /g}{\sqrt {(gt)^{2}-2gv_{0}\sin \theta t+v_{0}^{2}}}\,\mathrm {d} t.} Trajectories of a mass thrown at an angle of 70: without drag (a parabole) with Stokes' drag with Newtonian dragAir resistance creates a force that (for symmetric projectiles) is always directed against the direction of motion in the surrounding medium and has a magnitude that depends on the absolute speed: F a i r = f (v) v ^ {\displaystyle \mathbf {F_{air}} =-f(v)\cdot \mathbf {\hat {v}} } . The speed-dependence of the friction force is linear (f (v) v {\displaystyle f(v)\propto v}) at very low speeds (Stokes drag) and quadratic (f (v) v 2 {\displaystyle f(v)\propto v^{2}}) at large speeds (Newton drag).[8] The transition between these behaviours is determined by the Reynolds number, which depends on object speed and size, density {\textstyle \rho } and dynamic viscosity {\textstyle \eta } of the medium. For Reynolds numbers below about 1 the dependence is linear, above 1000 (turbulent flow) it becomes quadratic. In air, which has a kinematic viscosity / {\displaystyle \eta /\rho } around 0.15cm2/s, this means that the drag force becomes quadratic in v when the product of object speed and diameter is more than about 0.015 m2/s, which is typically the case for projectiles.Stokes drag: F a i r = k S t o k e s v {\displaystyle \mathbf {F_{air}} =-k_{\mathrm {Stokes} }\cdot \mathbf {v} \qquad } (for R e 1 {\textstyle Re\lesssim 1})Newton drag: F a i r = k | v | v {\displaystyle \mathbf {F_{air}} =-k\,|\mathbf {v} |\cdot \mathbf {v} \qquad } (for R e 1000 {\textstyle Re\gtrsim 1000})Free body diagram of a body on which only gravity and air resistance act.The free body diagram on the right is for a projectile that experiences air resistance and the effects of gravity. Here, air resistance is assumed to be in the direction opposite of the projectile's velocity: F a i r = f (v) v ^ {\displaystyle \mathbf {F_{\mathrm {air} }} =-f(v)\cdot \mathbf {\hat {v}} } Stokes drag, where F a i r v {\displaystyle \mathbf {F_{air}} \propto \mathbf {v} } , only applies at very low speed in air, and is thus not the typical case for projectiles. However, the linear dependence of F a i r {\displaystyle F_{\mathrm {air} }} on v {\displaystyle v} causes a very simple differential equation of motion d d t (v x v y) = (v x g v y) {\displaystyle {\frac {\mathrm {d} }{\mathrm {d} t}}{\begin{pmatrix}v_{x}\\v_{y}\end{pmatrix}}={\begin{pmatrix}-\mu \,v_{x}\\-g-\mu \,v_{y}\end{pmatrix}}} in which the 2 cartesian components become completely independent, and it is thus easier to solve.[9] Here, v 0 {\displaystyle v_{0}} , v x {\displaystyle v_{x}} and v y {\displaystyle v_{y}} will be used to denote the initial velocity, the velocity along the direction of x and the velocity along the direction of y, respectively. The mass of the projectile will be denoted by m, and := k / m {\displaystyle \mu :=k/m} . For the derivation only the case where 0 o 180 o {\textstyle 0°\leq \theta \leq 180°} is considered. Again, the projectile is fired from the origin (0,0).Derivation of horizontal positionThe relationships that represent the motion of the particle are derived by Newton's second law, both in the x and y directions. In the x direction F = k v x = m a x {\displaystyle \Sigma F=-kv_{x}=ma_{x}} and in the y direction F = k v y m g = m a y {\displaystyle \Sigma F=-kv_{y}-mg=ma_{y}} .This implies that: a x = v x = d v x d t {\displaystyle a_{x}=-\mu v_{x}={\frac {\mathrm {d} v_{x}}{\mathrm {d} t}}} (1),and a y = v y g = d v y d t {\displaystyle a_{y}=-\mu v_{y}-g={\frac {\mathrm {d} v_{y}}{\mathrm {d} t}}} (2)Solving (1) is an elementary differential equation, thus the steps leading to a unique solution for vx and, subsequently, x will not be enumerated. Given the initial conditions v x = v x 0 {\displaystyle v_{x}=v_{x0}} (where vx0 is understood to be the x component of the initial velocity) and x = 0 {\displaystyle x=0} for t = 0 {\displaystyle t=0} : v x = v x 0 e t {\displaystyle v_{x}=v_{x0}e^{-\mu t}} (1a) x (t) = v x 0 (1 e t) {\displaystyle x(t)={\frac {v_{x0}}{\mu }}\left(1-e^{-\mu t}\right)} (1b)Derivation of vertical positionWhile (1) is solved much in the same way, (2) is of distinct interest because of its non-homogeneous nature. Hence, we will be extensively solving (2). Note that in this case the initial conditions are used v y = v y 0 {\displaystyle v_{y}=v_{y0}} and y = 0 {\displaystyle y=0} when t = 0 {\displaystyle t=0} . d v y d t = v y g {\displaystyle {\frac {\mathrm {d} v_{y}}{\mathrm {d} t}}=-\mu v_{y}-g} (2) d v y d t + v y = g {\displaystyle {\frac {\mathrm {d} v_{y}}{\mathrm {d} t}}+\mu v_{y}=-g} (2a)This first order, linear, non-homogeneous differential equation may be solved a number of ways; however, in this instance, it will be quicker to approach the solution via an integrating factor e d t {\displaystyle e^{\int \mu \,\mathrm {d} t}} . e t (d v y d t + v y) = e t (g) {\displaystyle e^{\mu t}({\frac {\mathrm {d} v_{y}}{\mathrm {d} t}}+\mu v_{y})=e^{\mu t}(-g)} (2c) (e t v y) = e t (g) {\displaystyle (e^{\mu t}v_{y})^{\prime }=e^{\mu t}(-g)} (2d) (e t v y) d t = e t v y = e t (g) d t {\displaystyle \int {(e^{\mu t}v_{y})^{\prime }}\,\mathrm {d} t}=e^{\mu t}v_{y}=\int {e^{\mu t}(-g)\,\mathrm {d} t}} (2e) e t v y = 1 e t (g) + C {\displaystyle e^{\mu t}v_{y}={\frac {1}{\mu }}e^{\mu t}(-g)+C} (2f) v y = g + C e t {\displaystyle v_{y}={\frac {-g}{\mu }}+Ce^{-\mu t}} (2g)And by integration we find: y = g t 1 (v y 0 + g) e t + C {\displaystyle y=-{\frac {g}{\mu }}t-{\frac {1}{\mu }}(v_{y0}+{\frac {g}{\mu }})e^{-\mu t}+C} (3)Solving for our initial conditions: v y (t) = g + (v y 0 + g) e t {\displaystyle v_{y}(t)=-{\frac {g}{\mu }}+(v_{y0}+{\frac {g}{\mu }})e^{-\mu t}} (2h) y (t) = g t 1 (v y 0 + g) e t + 1 (v y 0 + g) {\displaystyle y(t)=-{\frac {g}{\mu }}t-{\frac {1}{\mu }}(v_{y0}+{\frac {g}{\mu }})e^{-\mu t}+{\frac {1}{\mu }}(v_{y0}+{\frac {g}{\mu }})} (3a)With a bit of algebra to simplify (3a): y (t) = g t + 1 (v y 0 + g) (1 e t) {\displaystyle y(t)=-{\frac {g}{\mu }}t+{\frac {1}{\mu }}\left(v_{y0}+{\frac {g}{\mu }}\right)(1-e^{-\mu t})} (3b)Derivation of the time of flightThe total time of the journey in the presence of air resistance (more specifically, when F a i r = k v {\displaystyle F_{air}=-kv}) can be calculated by the same strategy as above, namely, we solve the equation y (t) = 0 {\displaystyle y(t)=0} . While in the case of zero air resistance this equation can be solved elementarily, here we shall need the Lambert W function. The equation y (t) = g t + 1 (v y 0 + g) (1 e t) = 0 {\displaystyle y(t)=-{\frac {g}{\mu }}t+{\frac {1}{\mu }} (v_{y0}+{\frac {g}{\mu }})(1-e^{-\mu t})=0} is of the form c 1 t + c 2 + c 3 e c 4 t = 0 {\displaystyle c_{1}t+c_{2}+c_{3}e^{c_{4}t}=0} , and such an equation can be transformed into an equation solvable by the W {\displaystyle W} function (see an example of such a transformation here). Some algebra shows that the total time of flight, in closed form, is given as[10] t = 1 (1 + g v y 0 + W ((1 + g v y 0) e (1 + g v y 0))) {\displaystyle t={\frac {1}{\mu }}\left(1+{\frac {\mu }{g}}v_{y0}+W{\bigl (}-(1+{\frac {\mu }{g}}v_{y0})e^{-(1+{\frac {\mu }{g}}v_{y0})}{\bigr)}\right)} .Trajectories of a skydiver in air with Newton drag: para-flight; bomber near(ly above) target. v . d = 10 3 . = 150 {\textstyle v.d=10^{3}.u =150} cm2/s?The most typical case of air resistance, in case of Reynolds numbers above about 1000, is Newton drag with a drag force proportional to the speed squared, F a i r = k v 2 {\displaystyle F_{\mathrm {air} }=-kv^{2}} . In air, which has a kinematic viscosity around 0.15cm2/s, this means that the product of object speed and diameter must be more than about 0.015 m2/s.Unfortunately, the equations of motion can not be easily solved analytically for this case. Therefore, a numerical solution will be examined.The following assumptions are made:Constant gravitational acceleration g {\textstyle g} Air resistance is given by the following drag formula, F D = 1 2 c A v v {\displaystyle \mathbf {F_{D}} =-{\tfrac {1}{2}}c\rho A\,v\,\mathbf {v} } Where:FD is the drag force,c is the drag coefficient, is the air density,A is the cross sectional area of the projectile. Again = k / m := {\displaystyle \mu =k/m:=} c A / (2 m) {\displaystyle c\rho A/(2m)} = c / (2 p l) {\displaystyle =c\rho /(2\rho _{p}l)} . Compare this with theory/practice of the ballistic coefficient.Even though the general case of a projectile with Newton drag cannot be solved analytically, some special cases can. Here we denote the terminal velocity in free-fall as v = g / {\textstyle v_{\infty }={\sqrt {g/\mu }}} and the characteristic settling time constant t f 1 / (v) = 1 / g {\textstyle t_{f}\equiv 1/(v_{\infty }\mu)=1/{\sqrt {g\mu }}} . (Dimension of g {\displaystyle g} [m/s2], {\displaystyle \mu } [1/m])Near-horizontal motion: In case the motion is almost horizontal, | v x | | v y | {\displaystyle |v_{x}|\gg |v_{y}|} , such as a flying bullet. The vertical velocity component has very little influence on the horizontal motion. In this case:[11] v x (t) = v x 2 (t) {\displaystyle {\dot {v}}_{x}(t)=-\mu \,v_{x}^{2}(t)} v x (t) = 1 1 / v x , 0 + t {\displaystyle v_{x}(t)={\frac {1}{1/v_{x,0}+\mu \,t}}} x (t) = 1 ln (1 + v x , 0 t) {\displaystyle x(t)={\frac {1}{\mu }}\ln(1+\mu \,v_{x,0}\cdot t)} The same pattern applies for motion with friction along a line in any direction, when gravity is negligible (relatively small g {\displaystyle g}). It also applies when vertical motion is prevented, such as for a moving car with its engine off.Vertical motion upward:[11] v y (t) = g v y 2 (t) {\displaystyle {\dot {v}}_{y}(t)=-g-\mu \,v_{y}^{2}(t)} v y (t) = v tan t p e a k t t f {\displaystyle v_{y}(t)=v_{\infty }\tan {\frac {t_{\mathrm {peak} }-t}{t_{f}}}} y (t) = y p e a k + 1 ln (cos t p e a k t t f) {\displaystyle y(t)=y_{\mathrm {peak} }+{\frac {1}{\mu }}\ln {\bigl (}\cos {\frac {t_{\mathrm {peak} }-t}{t_{f}}}{\bigr)}} Here v g {\displaystyle v_{\infty }\equiv {\sqrt {\frac {g}{\mu }}}} and t f = 1 g , {\displaystyle t_{f}={\frac {1}{\sqrt {\mu g}}},} t p e a k t f arctan v y , 0 v = 1 g arctan (g v y , 0) , {\displaystyle t_{\mathrm {peak} }\equiv t_{f}\arctan {\frac {v_{y,0}}{v_{\infty }}}={\frac {1}{\sqrt {\mu g}}}\arctan {\left({\sqrt {\frac {\mu }{g}}}v_{y,0}\right)},} and y p e a k 1 ln cos t p e a k t f = 1 2 ln (1 + g v y , 0 2) {\displaystyle y_{\mathrm {peak} }\equiv -{\frac {1}{\mu }}\ln {\cos {\frac {t_{\mathrm {peak} }}{t_{f}}}}={\frac {1}{2\mu }}\ln {{\bigl (}1+{\frac {\mu }{g}}v_{y,0}^{2}{\bigr)}}} where v y , 0 {\displaystyle v_{y,0}} is the initial upward velocity at t = 0 {\displaystyle t=0} and the initial position is y (0) = 0 {\displaystyle y(0)=0} .A projectile cannot rise longer than t r i s e = 2 t f {\displaystyle t_{\mathrm {rise} }={\frac {\pi }{2}}t_{f}} in the vertical direction, when it reaches the peak (0 m, ypeak) at 0 m/s.Vertical motion downward:[11] v y (t) = g + v y 2 (t) {\displaystyle {\dot {v}}_{y}(t)=-g+\mu \,v_{y}^{2}(t)} v y (t) = v tanh t t p e a k t f {\displaystyle v_{y}(t)=-v_{\infty }\tanh {\frac {t-t_{\mathrm {peak} }}{t_{f}}}} y (t) = y p e a k 1 ln (cosh (t t p e a k t f)) {\displaystyle y(t)=y_{\mathrm {peak} }-{\frac {1}{\mu }}\ln(\cosh {\biggl (}{\frac {t-t_{\mathrm {peak} }}{t_{f}}}{\biggr)})} With hyperbolic functionsAfter a time t f {\displaystyle t_{f}} at y=0, the projectile reaches almost terminal velocity v {\displaystyle -v_{\infty }} .A projectile motion with drag can be computed generically by numerical integration of the ordinary differential equation, for instance by applying a reduction to a first-order system. The equation to be solved is d d t (x y v x v y) = (v x v y v x v x 2 + v y 2 g v y v x 2 + v y 2) {\displaystyle {\frac {\mathrm {d} }{\mathrm {d} t}}{\begin{pmatrix}x\\y\\v_{x}\\v_{y}\end{pmatrix}}= {\begin{pmatrix}v_{x}\\v_{y}\\-\mu \,v_{x}{\sqrt {v_{x}^{2}+v_{y}^{2}}}\\-g-\mu \,v_{y}{\sqrt {v_{x}^{2}+v_{y}^{2}}}\end{pmatrix}}} .This approach also allows to add the effects of speed-dependent drag coefficient, altitude-dependent air density (in product c (v) (y) {\displaystyle c(v)\rho (y)}) and position-dependent gravity field g (y) = g 0 / (1 + y / R) 2 {\textstyle g(y)=g_{0}/(1+y/R)^{2}} (when y R : g g 0 / (1 + 2 y / R) g 0 (1 2 y / R) {\textstyle y\ll R:g\lesssim g_{0}/(1+2y/R)\approx g_{0}(1-2y/R)} , is linear decrease).Lofted trajectories of North Korean ballistic missiles Hwasong-14, Hwasong-15 and Hwasong-17A special case of a ballistic trajectory for a rocket is a lofted trajectory, a trajectory with an apogee greater than the minimum-energy trajectory to the same range. In other words, the rocket travels higher and by doing so it uses more energy to get to the same landing point. This may be done for various reasons such as increasing distance to the horizon to give greater viewing/communication range or for changing the angle with which a missile will impact on landing. Lofted trajectories are sometimes used in both missile rocketry and in spaceflight.[12]Projectile trajectory around a planet, compared to the motion in a uniform gravity fieldWhen a projectile travels a range that is significant compared to the Earth's radius (above 100km), the curvature of the Earth and the non-uniform Earth's gravity have to be considered. This is, for example, the case with spacecrafts and intercontinental missiles. The trajectory then generalizes (without air resistance) from a parabola to a Kepler-ellipse with one focus at the center of the Earth (shown in fig. 3). The projectile motion then follows Kepler's laws of planetary motion.The trajectory's parameters have to be adapted from the values of a uniform gravity field stated above. The Earth radius is taken as R, and g as the standard surface gravity. Let v ~ := v / R g {\displaystyle {\tilde {v}}:=v/{\sqrt {Rg}}} be the launch velocity relative to the first cosmic or escape velocity.Total range d between launch and impact: d = v 2 sin (2) g / 1 (2 v ~ 2) v ~ 2 cos 2 {\displaystyle d={\frac {v^{2}\sin(2\theta)}{g}}{\Big /}{\sqrt {1-\left(2-{\tilde {v}}^{2}\right){\tilde {v}}^{2}\cos ^{2}\theta }}} (where launch angle = 1 2 arccos (v ~ 2 / (2 v ~ 2)) {\displaystyle \theta ={\tfrac {1}{2}}\arccos \left({\tilde {v}}^{2}/(2-{\tilde {v}}^{2})\right)})Maximum range of a projectile for optimum launch angle =45o: d m a x = v 2 g / (1 1 2 v ~ 2) {\displaystyle d_{\mathrm {max} }={\frac {v^{2}}{g}}{\big /}\left(1-{\tfrac {1}{2}}{\tilde {v}}^{2}\right)} with v < R g {\displaystyle v