Last Updated on June 1, 2025 by Muhamed Elmesery Bernoulli equation, or the incompressible steady flow energy equation, is considered one of the most well-known equations in physics (fluid mechanics) and it explains the conservation of mechanical work-energy. The equation was published in 1738 by Daniel Bernoulli (a Swiss physicist) to help us understand fluid flow. Daniel Bernoulli was born on February 8th, 1700, in the Netherlands. He studied mathematics and medicine under the guidance of his father Johann Bernoulli. In this article, we will discuss Bernoulli equation, principle, derivation, application and more. Get started Praxilabs for FREE Bernoulli Equation P is the static pressure (the pressure of the fluid). ρ is the density. V is the velocity. G is the gravitational acceleration. H is the elevation of the fluid. Bernoulli's theorem describes the relation between velocity, pressure, and elevation of a flowing fluid in a streamline. It states that the dynamic pressure plus the static pressure in the flow, one half of the density (r) times the velocity (V) squared, is equal to a constant throughout the flow. The constant is called the total pressure of the flow (pt). Or simply, the Bernoulli equation states that the sum of the potential, kinetic, and flow energies of a fluid is constant in a streamline in steady flow. Note: The streamline in steady flow is the path traced by a single particle within the fluid. The following figure shows the streamline in steady flow: To apply Bernoulli's equation: The flow must be steady (Velocity, density, and pressure must not change at any point in the streamline). The flow must be incompressible: Even when the pressure varies, the density must remain constant along the streamline. Friction by viscous forces must be negligible or minimal. Note: Incompressible flows are gasses and gasses with a low Mach number and constant fluid density, regardless of the pressure flow. By using incompressible flow, we will have the simplest form of Bernoulli's equation. Bernoulli Differential Equations A Bernoulli differential equation is a specific type of ordinary differential equation that can be expressed in the form: y′+P(x)y=Q(x)yn Where: n is a real number (except for 0 or 1), because it is notable for its non-linear nature and if n=0 or n=1 then the equation is linear. Bernoulli equation for V In Bernoulli's equation, we can find the velocity of the fluid Where: ρ is the density of the fluid. P is the measured pressure. What is Bernoulli's Principle ? Bernoulli's principle states that an increase in the velocity (the speed of a fluid) occurs simultaneously and must be accompanied by a decrease in the potential energy of the fluid (or the static pressure). Bernoulli's principle can be derived from the conservation of energy principle. In case of steady flow, the sum of energy forms in a fluid will remain the same at all points of that streamline. While all the energy remains constant, an increase in the fluid velocity will imply that there is an increase in the kinetic energy or dynamic pressure. This happens with a decrease in the potential energy (static pressure and internal energy). Bernoulli's Principle Formula Bernoulli's equation formula is considered a relation between pressure, potential energy, and kinetic energy of a fluid. The formula is given as follows: The Bernoulli equation can be considered to be the conservation of energy principle for the flowing fluids. Bernoulli effect is the lowering of fluid pressure in cases where the velocity is increased. In the case of high velocity flow through the constriction, the kinetic energy must increase at the expense of pressure energy. Limitations on the Use of Bernoulli Theorem The Bernoulli equation is applicable to steady flow as we explained before, but in some cases we can't use the Bernoulli equation. The Bernoulli theorem is not applicable in a flow section like machines (e.g., fan or turbine or pump), because these machines can damage the streamlines and make energy interactions with the fluid. So in these cases, the energy equation should be used instead of Bernoulli equation. The equation also should not be used for significant temperature change flow sections like cooling or heating sections, because the gas density is inversely proportional to the temperature. Every flow involves some friction and if the frictional effects are not negligible, the Bernoulli theorem is not applicable in this case. So, the Bernoulli equation should not be used in the flow of: Sudden expansion. Heating section. Valve. Fan. Narrow long tube. Bernoulli Equation Derivation We can drive Bernoulli differential equation as follows: We will consider that we have a pipe with a varying height and diameter through which an incompressible fluid is flowing. The relationship between the areas of cross-sections A, pressure p the speed of the flow v, and the height h at two different points 1 and 2 is described in the following figure: Assumptions: The incompressible fluid density remains constant at both points. The fluid energy is conserved because there are no viscous forces in the fluid. So, the work done on the fluid is given as: dW = F1dx1 – F2dx2 dW = p1A1dx1 – p2A2dx2 dW = p1dv – p2dv = (p1 – p2)dv We know that the work done on the fluid was due to the conservation of change in both the kinetic energy and gravitational potential energy The change in kinetic energy of the fluid is given as: The change in potential energy is given as: dU = m2gy2 – m1gy1 = pdvg(y2 – y1) Then, the energy equation is given as: dW = dK + dU And finally by rearranging the equation, we get For more understanding, watch the following video about Bernoulli equation derivation PraxiLabs virtual science labs enable you to conduct various laboratory experiments in physics, chemistry, and biology online anytime and anywhere. Create your free account and try our virtual labs now. Book FREE Live Demo Now Bernoulli Equation Applications Bernoulli's equation has many applications. For example, we can use it to explain how planes generate lift, to calculate the velocity of fluid flow, or why ships have to run away from each other as they pass, and other applications that we find in our daily lives. We will go over in detail some of the most popular applications of Bernoulli's theorem. Bernoulli's equation is used to solve some fluid flow problems. The Bernoulli equation can be applied to devices such as the orifice meter, venturi meter, and Pitot tube and its applications for measuring flow in open channels and inside tubes. Venturi meter is a device that is based on Bernoulli's principle and used to determine the rate of flow through a pipe. It works by measuring the pressure drop across a section near the pipe. For an incompressible fluid, reducing the diameter will increase the fluid flow velocity. Note: Bernoulli's principle states that there must be a pressure drop in the region of the reduced diameter. This phenomenon is known as the venturi effect. Venturi meter As we mentioned before, the speed of a fluid flow can be measured by devices like Venturi meter, which is used to reduce the diameter of the flow and placed into a pipeline. The maximum possible discharge rate for a tank with an opening or a spout at the base can be calculated directly by Bernoulli's equation, and it has been found to be proportional to the square root of the height of the liquid in the tank (this is Torricelli's law) and this shows that Torricelli's law is compatible with Bernoulli's principle. Viscosity reduces the rate of discharge. How aircraft wings generate lift can be explained by Bernoulli's equation for fluids. In the case of flight, the fluid flowing above the wing of the aircraft moves faster than the fluid under the wing, and according to Bernoulli's principle this causes the creation of an area of high pressure under the plane and a region of low pressure above the plane. So, we can explain the reason for the wings and their upward curvature (according to Bernoulli's theory) that the air passes at a higher speed on the upper surface than the lower one. The difference in air velocity is calculated using Bernoulli's principle of difference in pressure. In some cases of fluid flows, we know the velocities at two points of the streamline and pressure at only one point. The unknown is the pressure at the other point of the fluid. So in this case (if the Bernoulli equation applies to it) we can use Bernoulli's Equation to find the unknown pressure. In cases where the elevation and pressure at two points of the streamline are known and velocity at one point is also known, and we want to find the unknown velocity. Bernoulli's equation is applied to calculate and find the required velocity. For example the flow through a siphon In this case, we need to find the velocity with which the fluid leaves the siphon. By applying Bernoulli's Equation between the reservoir surface and the siphon exit where the fluid leaves the siphon, the pressure at both points is the same and equal to the atmospheric pressure, and the velocity at the reservoir is negligible because the reservoir is large. We can calculate the velocity at the exit point by using the values of elevation at the two points. In echocardiography, Bernoulli's equation can be applied when interpreting blood flow to determine a localized pressure decrease produced by high flow rate near blockages. For clinical medicine, the equation is used for an easy estimation of pressure gradients from velocity. Bernoulli's equation can also be used in the venturi mask. The venturi mask is a medical oxygen device that delivers oxygen concentration to patients on controlled oxygen therapy. The mask has a tube that is connected to a nozzle which connects to a supply of pure oxygen. The tube that is directly connected to the mask has a small window which allows room air to flow into the mask. The venturi mask can control the amount of oxygen that flows in, along with pure oxygen delivered from its connected nozzle. Venturi masks use the Bernoulli principle, As oxygen flows into the tube, it creates a decrease in pressure due to oxygen passing through a narrow opening of the tube. The air is allowed to flow into the mask for the drop in pressure, mixing with the pure oxygen delivered from the nozzle, which is the consequence of Bernoulli's principle. Other Applications of Bernoulli's Principle Bernoulli's theory is used to study the unsteady potential flow used in the theory of ocean surface waves and acoustics. When we stand at a railway station and a train comes in, we tend to fall towards the train. We can explain this using Bernoulli's principle. As the train passes, the speed of the air between us and the train increases. Hence, from the equation, we can say that the pressure is decreasing. So the pressure from the back pushes us towards the train and that depends on the Bernoulli effect. The Bernoulli equation also explains how a Bunsen burner works. When the gas valve is opened, the gas flows into the barrel at a high velocity. According to Bernoulli's theory, this high velocity creates a low pressure area in the barrel (which draws air through the regulator) allowing the gas to burn more efficiently. Many flow meters are based on Bernoulli's principle to determine the velocity of a flowing fluid. The most famous of these devices is the Pitot-Static Tube, which is used in aircraft to measure flight speed. Bernoulli's principle also applies to swinging a cricket ball. During the match, players constantly polish one side of the ball. After some time, one side is completely rough and the other is still smooth. Hence, when the ball is thrown in the air, the velocity on one side of the ball is greater than the other due to its difference in smoothness. And this results in a pressure difference between the two sides. This causes the ball to spin ("swing") as it travels through the air. Bernoulli Equation Assumptions For Bernoulli's equation to be applied, the following assumptions must be met: The flow must be steady. (Velocity, pressure and density cannot change at any point). The flow must be incompressible – even when the pressure varies, the density must remain constant along the streamline. Friction by viscous forces must be minimal. Example of Bernoulli Differential Equations Try to solve the Bernoulli differential equation: y′ + y = xy2 Solution: Rewrite the equation in standard form: y′+y = xy2 Divide through by y2: \frac{y'}{y^2} + \frac{y}{y^2} = x Simplified: y^{-2}y' + y^{-1} = x Substitute v = y^{-1}: Derivative: \frac{dv}{dx} = -y^{-2} \frac{dy}{dx} Rewrite the original equation: -\frac{dv}{dx} + v = x Rearrange: \frac{dv}{dx} – v = -x Find the integrating factor: \mu(x) = e^{\int -1 \, dx} = e^{-x} Multiply through by the integrating factor: e^{-x} \frac{dv}{dx} – e^{-x} v = -xe^{-x} Simplify: \frac{d}{dx} \left(e^{-x} v \right) = -xe^{-x} \int -xe^{-x} \, dx = C Using integration by parts: \int -xe^{-x} \, dx = -xe^{-x} – \int e^{-x} \, dx = -xe^{-x} + e^{-x} Simplify: e^{-x} v = xe^{-x} + e^{-x} + C v = x + 1 + Ce^{x} Back-substitute v = y^{-1}: y^{-1} = x + 1 + Ce^{x} Solve for y: y = \frac{1}{x + 1 + Ce^{x}} Source: How to Solve Bernoulli Differential Equation Gamified learning experiences with Bernoulli's equation Gamification is the method that depends on using game elements and game mechanisms in non-game contexts. Some researchers suggest that it could also be used in web-based education as a tool to increase student motivation and engagement. For example, using gamified learning elements in teaching Bernoulli's equation can motivate and encourage learners to participate in the learning process and thus improve their learning outcomes and also enhance their engagement and motivation. In an attempt to verify those theories, researchers conducted a study "Gamifying Learning Experiences: Practical Implications and Outcomes" they designed and developed a gamification plugin for a well-known e-learning platform. They conducted an experiment using this plugin in a university course, collecting quantitative and qualitative data in the process. Their findings suggest that some common beliefs about the benefits obtained when using games in education can be challenged. Students who completed the gamified experience scored better in practical assignments and achieved higher overall scores, but their findings also suggest that these students performed poorly on written assignments and participated less in class activities, although their initial motivation was higher. Study Interactive simulations of Bernoulli's equation Virtual lab simulations can enrich the experimentation process, help students to understand Bernoulli's equation. Educational virtual labs for fluid mechanics such as the interactive simulations of Bernoulli's equation simulate the water flowing through a pipe. You can adjust the outlet height, outlet pipe diameter, inlet pressure, and inlet velocity using the sliders. Virtual labs for Online experiments with fluid flow phenomena provide a wide range of interactive features: Interactive 3D virtual labs provide students with tools that allow for more focus, attention, and engagement. Students stay interested, alert, and absorb information more easily. Virtual labs eliminate potential safety hazards associated with handling hazardous materials or conducting experiments that require specialized equipment. Students can learn and experiment in a safe environment, reducing the chances of accidents or mishaps. By using virtual labs, you can save time and effort, as they eliminate the need to move between different laboratories. Students can access experiments and learning resources without the need for setup or cleanup, allowing them to focus more on the core learning objectives. Virtual labs provide a virtual learning and teaching environment that aims to develop practical skills and learning outcomes. Students can conduct experiments with 24/7 unlimited accessibility and repeat virtual experiments as many times as needed until they grasp and understand all the information. Since virtual simulations are accessible via the Internet, students can perform numerous experiments without being confined to specific locations or times, unlike when using real laboratories. PraxiLabs simulation lab enable you to conduct various laboratory experiments in physics, chemistry, and biology online anytime and anywhere. Create your free account and try PraxiLabs virtual labs now. Join our virtual physics lab and start your virtual journey Bernoulli's equation is a simple but incredibly important equation in physics and engineering that can help us understand a lot about the behavior of fluids. It describes the relationship between the pressure, velocity and elevation of a flowing fluid.You can watch the video below for an animated introduction to Bernoulli's equation, or just keep reading to learn more. equation is essentially a statement of the conservation of energy. It tells us that the static pressure (how much the fluid is pressurised), the dynamic pressure (how fast the fluid is moving), and the hydrostatic pressure (how high up the fluid is) of the fluid remain constant along a streamline.Here is the equation, which was first published by Daniel Bernoulli in 1738.The equation reflects the idea that energy is conserved along a streamline because the three terms can be thought of a representing the pressure energy, kinetic energy and potential energy of the fluid. Bernoulli's equation states that the overall sum of these energies doesn't change along a streamline – the energy of the fluid is just transferring between these different forms. If the elevation of the fluid increases, for example, the pressure or the velocity must reduce in proportion.In the real world energy isn't completely conserved – some energy will always dissipate as a fluid flows, because of the viscosity of the fluid. Energy conservation is one of several assumptions that is built into Bernoulli's equation. We'll cover some of the other assumptions later on.What is a streamline?Bernoulli's equation states that the static, dynamic and hydrostatic pressures are constant along a streamline. But what's a streamline?In steady flow (flow that doesn't vary with time), a streamline is defined as the path traced by a single particle within the fluid.Bernoulli's equation is usually applied to compare the flow conditions between two points on the same streamline, in which case it is written like this:$$P_1 + \frac{\rho_1}{2}{V_1}^2 + \rho_1 g h_1 = P_2 + \frac{\rho_2}{2}{V_2}^2 + \rho_2 g h_2$$Bernoulli's equation helps us understand how the pressure, velocity and elevation of a fluid are linked. We can use it to calculate the velocity of water exiting a hose, or to figure out if there is a risk of cavitation (low pressure) in a piping system, for example. It does make some simplifying assumptions, but it is a very powerful tool.The Efficient Engineer Summary SheetsThe Efficient Engineer summary sheets are designed to present all of the key information you need to know about a particular topic on a single page. It doesn't get more efficient than that!Get The Summary Sheets!Let's look at an example where we apply Bernoulli's equation to flow through a restriction in a water hosepipe. The objective is to calculate the change in pressure between Point 1 and Point 2, which are on the same streamline.$$P_1 + \frac{\rho_1}{2}{V_1}^2 + \rho_1 g h_1 = P_2 + \frac{\rho_2}{2}{V_2}^2 + \rho_2 g h_2$$In this example we'll apply Bernoulli's equation to calculate the change in pressure at the restriction in this hosepipeThe difference in elevation between the two points is negligible ($h_1 = h_2$), so we can remove the elevation terms from Bernoulli's equation, and re-arrange obtain this equation for the difference in pressure between the two points.$$P_1 – P_2 = \frac{\rho}{2}({{V_2}^2 – V_1}^2)$$The fluid is incompressible, so the mass flow rate between points 1 and 2 must be the same:$$\dot{m}_1 = \dot{m}_2$$$$\rho A_1 V_1 = \rho A_2 V_2$$By re-arranging we can see that the velocity at point 2 is equal to the velocity at point 1 multiplied by the ratio of the areas of the pipe at the two points.$$V_2 = \frac{A_1}{A_2}V_1$$Substituting this into the pressure difference equation gives us an equation for the pressure change based only on V_1, A_1, A_2 and the fluid density ρ.$$P_1 – P_2 = \frac{\rho}{2} {V_1}^2(\frac{{A_1}^2}{{A_2}^2}-1)$$Because of the restriction $A_2 < A_1$, which means that the velocity increases between Point 1 and Point 2 but the pressure decreases.The Venturi meter is a flow measurement device that works on this principle – it measures the pressure drop across a converging section of pipe ($P1 – P2$), and applies Bernoulli's equation to calculate the fluid flow rate.A Venturi meter measures the pressure drop across a converging section of pipe and applies Bernoulli's equation to calculate the fluid flow rateAs we saw in the example above, Bernoulli's equation tells us that if you restrict the flow of water through a hosepipe, the velocity of the water will increase as it passes through the restriction, but the pressure of the water in the restricted section will drop.This might seem counter-intuitive – it feels natural to assume that an increase in velocity results in an increase in pressure, but it makes sense when you think about the conservation of energy. The energy needed to increase the velocity of the fluid comes at the expense of the pressure energy.This is an interesting observation that we can derive from Bernoulli's equation – for horizontal flow (i.e. when there is no change in the elevation term) if there is an increase in the velocity of the fluid, there must be a decrease in pressure. This observation is known as Bernoulli's Principle. For horizontal flow an increase in velocity must be accompanied by a decrease in pressure.Bernoulli's equation can be stated in three different forms, depending on whether each term has units of energy, pressure or distance. The form of the equation shown at the top of this page was the pressure form.$$\frac{P}{\rho} + \frac{V^2}{2} + gh = \mathrm{constant}$$$$P + \frac{\rho}{2}V^2 + \rho gh = \mathrm{constant}$$$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \mathrm{constant}$$These forms of Bernoulli's equation are saying the same thing – that energy is conserved along a streamline. They are just using different quantities to express this concept. Which form of the equation you should use depends on what it is you are trying to calculate (energy, pressure or head).Bernoulli's based on a few assumptions that it's important to understand to be able to apply it correctly:Inviscid flow – Bernoulli's equation assumes that the fluid is inviscid, meaning that it has no viscosity. If the fluid is viscous there would be some dissipation of the fluid's internal energy as it flows, and the idea that energy is conserved along a streamline would not apply. Of course no fluid is truly inviscid, so some judgement needs to be used when deciding if the equation is applicable.Laminar and steady flow – Bernoulli's equation can only be used if the flow is steady (i.e. it doesn't vary with time), and it is laminar. The equation is not applicable for turbulent flow. We've got a page covering laminar and turbulent flow if you'd like to learn more about these two flow regimes.Incompressible flow – Bernoulli's equation assumes that the fluid is incompressible. This assumption is usually valid for liquids, but might not be for gases, particularly at high velocities. Laminar and turbulent flow are two fundamentally different flow regimes that can be observed in a flowing fluid.Learn more Viscosity is a measure of the resistance of a fluid to flow, and is defined as the ratio of shear stress to the rate of shear strain in a fluid,Learn more Bernoulli's equation is a simple but incredibly important equation in physics and engineering that can help us understand a lot about the behavior of fluids. 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A special form of the Euler's equation derived along a fluid flow streamline is often called the Bernoulli Equation: Energy Form For steady state in-compressible flow the Euler equation becomes E = p1 / ρ + v12 / 2 + g h1 = p2 / ρ + v2 2 / 2 + g h2 - Eloss = constant (1) where E = energy per unit mass in flow (J/kg, Btu/slug) p = pressure in the fluid (Pa, psf) ρ = density of fluid (kg/m3, slug/ft3) v = velocity of fluid (m/s, ft/s) Eloss = energy lost per unit mass in flow (J/kg, Btu/slug) 1 psi = 144 psf Head Form (1) can be modified by dividing with gravity like h = p1 / γ + v12 / (2 g) + h1 = p2 / γ + v2 2 / (2 g) + h2 - Eloss / g = constant (2) where h = head (m fluid column, ft fluid column) γ = ρ g = specific weight of fluid (N/m3, lbf/ft3) Equation (2) is often referred to as the "head" because all elements has the unit of length. Note! - the head unit is with reference to the density of the flowing fluid. For other units - like mm Water Column - check Velocity Pressure Head. Dynamic Pressure (1) and (2) are two forms of the Bernoulli Equation for a steady state in-compressible flow. If we assume that the gravitational body force is negligible - the elevation is small - then the Bernoulli equation can be modified to p = p1 + ρ v12 / 2 = p2 + ρ v22 / 2 - ploss = p1 + pd1 = p2 + pd2 - ploss (3) where p = pressure (Pa, psf) ploss = pressure loss (Pa, psi) pd = 1/2 ρ v2= dynamic pressure (Pa, psf) It is common to refer to the flow velocity component as the dynamic pressure of the fluid flow. Note! - increased flow velocity reduces pressure - decreased flow velocity increases pressure. This phenomena can be observed in a venturi meter where the pressure is reduced in the constriction area and regained after. It can also be observed in a pitot tube where the stagnation pressure is measured. The stagnation pressure is where the velocity component is zero. Discharge from a Pressure Vessel - Bernoulli Equation and Flow from a Tank through a small Orifice Liquid flows from a tank through a orifice close to the bottom. The Bernoulli equation can be adapted to a streamline from the surface (1) to the orifice (2) : p1 / γ + v12 / (2 g) + h1 = p2 / γ + v22 / (2 g) + h2 - Eloss / g (4) By multiplying with g and assuming that the energy loss is neglect-able - (4) can be transformed to p1 / ρ + v12 / 2 + g h1 = p2 / ρ + v2 2 / 2 + g h2 (4b) Discharge Velocity If h = h1 - h2 (4c) and (according the continuity equation) v1 = (A2 / A1) v2 (4d) then the velocity out of the orifice can be expressed as v2 = (2 / (1 - A2 / A1)2) (g h + (p1 - p2) / ρ))1/2 (5) Vented Tank For a vented tank where the inside pressure equals the outside pressure p1 = p2 (5b) and the surface area is much larger than the orifice area A1 >> A2 (5c) - then eq. 5 can be modified to v2 = (2 g h)1/2 (6) "The velocity out from the tank is equal to speed of a freely body falling the distance h ." - also known as Torricelli's Theorem. Example - outlet velocity from a vented tank The outlet velocity from a tank with level 10 m can be calculated as v2 = (2 (9.81 m/s2) (10 m))1/2 = 14 m/s Orifice Discharge Coefficient Eq. 6 is for ideal flow without pressure loss in the orifice. In the real worlds - with pressure loss - eq. 6 can be expressed with a coefficient of discharge - friction coefficient - as v2 = c (2 g h)1/2 (6b) where c = coefficient of discharge The coefficient of discharge can be determined experimentally. For a sharp edged opening it may be as low as 0.6. For smooth orifices it may be between 0.95 and 1 . Pressurized Tank If the tank is closed, pressurized and the level between the surface and the discharge outlet minimal (the influence from level difference is very small compared to pressure influence in eq. 5) - the discharge velocity can be expressed as v2 = c (2 (p1 - p2) / ρ)1/2 (7) Draining Containers or Tanks - Volume Flow and Emptying Time Calculator Example - Outlet Velocity from a Pressurized Tank The outlet velocity of a pressurized tank where p1 = 0.2 (MN/m2, 106 Pa) p2 = 0.1 (MN/m2, 106 Pa) A2 / A1 = 0.01 h = 10 (m) can be calculated as V2= ((2 / (1 - (0.01)2)) ((0.2×106 N/m2) - (0.1×106 N/m2)) / (1000 kg/m3) + (9.81 m/s2) (10 m)))1/2 = 19.9 m/s Energy Loss through a Reduction Valve When fluid flows through a valve, the pressure is reduced - there is an energy loss. By neglecting the change in elevation (h1 = h2) and the change in fluid velocity (v1 = v2) the pressure energy loss between before the valve and the pressure after the valve including the energy loss through the valve - is constant. The Bernoulli equation can be modified to p1 / ρ = p2 / ρ + Eloss (8) where Eloss = energy loss through valve (J) (8) can be transformed to: Eloss = (p1 - p2) / ρ (8b) Energy is the capacity to do work. The hydraulic grade line and the energy line are graphical presentations of the Bernoulli equation. Equations used in fluid mechanics - like Bernoulli, conservation of energy, conservation of mass, pressure, Navier-Stokes, ideal gas law, Euler equations, Laplace equations, Darcy-Weisbach Equation and more. The Equation of Continuity is a statement of mass conservation. An introduction to the different types of fluid flowmeters - Orifices, Venturies, Nozzles, Rotameters, Pitot Tubes, Calorimetrics, Turbine, Vortex, Electromagnetic, Doppler, Ultrasonic, Thermal, Coriolis. Calculate liquid velocity, volume flow and emptying time when emptying a container. The Mechanical Energy Equation compared to the Extended Bernoulli Equation. Pitot tubes can be used to measure fluid flow velocities by measuring the difference between static and dynamic pressure in the flow. Sluice gates can be used to control and measure volume flow rates in open channels and rivers, mainly in connection to hydro power plants. Steam flow through orifices - for steam pressures ranging 2 - 300 psi Steam leaks through orifices - for pressures ranging 250 - 600 psi In the realm of fluid dynamics, Bernoulli's Equation stands as a cornerstone principle, pivotal to understanding and predicting the behavior of fluid flow in various engineering applications. Named after the Swiss mathematician Daniel Bernoulli, this equation provides a relationship between the pressure, velocity, and elevation in a moving fluid. Its significance extends across numerous fields, including aerospace, civil, and mechanical engineering, making it an indispensable tool for engineers and scientists alike. This article delves into the fundamentals, historical development, practical applications, advanced topics, and challenges associated with Bernoulli's Equation, offering a comprehensive overview of its role in fluid dynamics. Fundamentals Basic Principles and Concepts Bernoulli's Equation is derived from the principle of conservation of energy, specifically applied to fluid flow. It states that for an incompressible, non-viscous fluid, the total mechanical energy along a streamline remains constant. The equation is expressed as: P + 0.5ρv² + ρgh = constant Where: P = Pressure energy per unit volume ρ = Fluid density v = Fluid velocity g = Acceleration due to gravity h = Elevation height This equation implies that an increase in the fluid's velocity leads to a decrease in its pressure or potential energy, and vice versa. The assumptions underlying Bernoulli's Equation include steady flow, incompressibility, and negligible viscosity, which simplify the complex nature of real-world fluid dynamics. Key Terms Streamline: A path followed by fluid particles under steady flow conditions. Incompressible Fluid: A fluid whose density remains constant regardless of the pressure changes. Non-viscous Fluid: An ideal fluid with no internal friction (viscosity). Dynamic Pressure: The kinetic energy per unit volume of a fluid particle, represented as 0.5ρv². Static Pressure: The pressure exerted by a fluid at rest or in motion, excluding dynamic pressure. Historical Development The origins of Bernoulli's Equation can be traced back to the early 18th century. Daniel Bernoulli, a prominent mathematician and physicist, published his seminal work "Hydrodynamica" in 1738, where he introduced the principle that would later bear his name. Bernoulli's insights were built upon the foundational work of earlier scientists such as Isaac Newton and Leonhard Euler, who made significant contributions to the understanding of fluid mechanics. One of the key milestones in the development of Bernoulli's Equation was the formalization of the conservation of energy principle in fluid flow. Euler, a contemporary of Bernoulli, extended these ideas and formulated the Euler equations, which describe the motion of an inviscid fluid. These equations laid the groundwork for the modern interpretation and application of Bernoulli's principle. Over the centuries, Bernoulli's Equation has been refined and validated through experimental and theoretical advancements, solidifying its status as a fundamental principle in fluid dynamics. Applications Aerospace Engineering In aerospace engineering, Bernoulli's Equation is instrumental in understanding the lift generated by aircraft wings. The shape of an airfoil causes the air to travel faster over the top surface than the bottom surface, resulting in lower pressure above the wing and higher pressure below. This pressure difference creates lift, enabling the aircraft to fly. Bernoulli's principle is also applied in the design of jet engines and wind tunnels. Civil Engineering Civil engineers utilize Bernoulli's Equation in the design and analysis of water supply systems, sewage networks, and hydraulic structures. For instance, the equation helps determine the pressure distribution and flow rates in pipelines, ensuring efficient water delivery and waste removal. It is also used in the design of dams and spillways to predict water flow behavior and prevent structural failures. Mechanical Engineering In mechanical engineering, Bernoulli's Equation is applied in the design of various fluid machinery, such as pumps, turbines, and compressors. It aids in predicting the performance and efficiency of these devices by analyzing the energy transformations within the fluid. Additionally, the equation is used in HVAC (heating, ventilation, and air conditioning) systems to optimize airflow and pressure distribution. Case Study: Venturi Meter A practical application of Bernoulli's Equation is the Venturi meter, a device used to measure the flow rate of fluids in a pipeline. The Venturi meter consists of a converging section, a throat, and a diverging section. As the fluid flows through the converging section, its velocity increases, causing a decrease in pressure at the throat. By measuring the pressure difference between the inlet and the throat, the flow rate can be determined using Bernoulli's Equation. Advanced Topics Compressible Flow While Bernoulli's Equation is primarily applicable to incompressible fluids, it can be extended to compressible flow scenarios with certain modifications. In compressible flow, changes in fluid density must be accounted for, leading to more complex equations such as the compressible Bernoulli's Equation. This extension is crucial in high-speed aerodynamics, where air density variations significantly impact the behavior of the flow. Viscous Effects Real-world fluids exhibit viscosity, which introduces frictional forces and energy dissipation. To account for these effects, the Navier-Stokes equations are used in conjunction with Bernoulli's Equation. These equations describe the motion of viscous fluids and are essential for accurately modeling fluid behavior in engineering applications such as lubrication, blood flow, and polymer processing. Recent Research and Innovations Recent advancements in computational fluid dynamics (CFD) have enabled more precise simulations of fluid flow, incorporating Bernoulli's principles with complex boundary conditions and turbulence models. Innovations in sensor technology and data analytics have also enhanced the accuracy of flow measurements and diagnostics, leading to improved designs and performance in various engineering systems. Assumptions and Limitations One of the primary challenges associated with Bernoulli's Equation is its reliance on simplifying assumptions, such as incompressibility and negligible viscosity. These assumptions may not hold true in all scenarios, leading to discrepancies between theoretical predictions and real-world observations. Engineers must carefully evaluate the applicability of Bernoulli's Equation and consider alternative models when necessary. Complex Flow Patterns In practical applications, fluid flow often exhibits complex patterns, including turbulence, separation, and secondary flows. These phenomena are not adequately captured by Bernoulli's Equation alone, necessitating the use of advanced computational methods and experimental techniques to obtain accurate results. Potential Solutions and Future Research To address these challenges, ongoing research focuses on developing more comprehensive models that integrate Bernoulli's principles with advanced fluid dynamics theories. Enhanced computational capabilities and experimental techniques are also being explored to improve the accuracy and reliability of flow predictions. Future research may uncover new insights into fluid behavior, leading to innovative engineering solutions and applications. Conclusion Bernoulli's Equation remains a fundamental principle in the field of fluid dynamics, offering valuable insights into the behavior of fluid flow in various engineering applications. From aerospace to civil and mechanical engineering, its applications are vast and diverse, underscoring its importance in designing efficient and effective systems. While challenges and limitations exist, ongoing research and advancements continue to refine and expand our understanding of fluid dynamics, ensuring that Bernoulli's legacy endures in the ever-evolving landscape of engineering. By the end of this section, you will be able to: Explain the terms in Bernoulli's equation. Explain how Bernoulli's equation is related to conservation of energy. Explain how to derive Bernoulli's principle from Bernoulli's equation. Calculate with Bernoulli's principle. List some applications of Bernoulli's principle. When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem, [latex]{W}_{\text{net}}=\frac{1}{2}{mv}^{2}-\frac{1}{2}{{mv}_{0}}^{2}\\[/latex]. There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the pressure will drop in a rapidly-moving fluid, whether or not the fluid is confined to a tube. There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See Figure 1.) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another. Figure 1. An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v2 is greater than v1), causing the pressure between them to drop (Pi is less than Po). Greater pressure on the outside pushes the car and truck together. Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior. The relationship between pressure and velocity in fluids is described quantitatively by Bernoulli's equation, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant: [latex]P+\frac{1}{2}\rho v^{2}+\rho gh=\text{constant}\\[/latex], where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes [latex]{P}_{1}+\frac{1}{2}{{\rho v}_{1}}^{2}+\rho {gh}_{1}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}+\rho {gh}_{2}\\[/latex]. Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ. In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting ρ = m/V into it and gathering terms: [latex]\frac{1}{2}{\rho v}^{2}=\frac{\frac{1}{2}{\text{mv}}^{2}}{V}=\frac{\text{KE}}{V}\\[/latex] So [latex]\frac{1}{2}{\rho v}^{2}\\[/latex] is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find [latex]\rho {gh}=\frac{mgh}{V}=\frac{{\text{PE}}_{\text{g}}}{V}\\[/latex], so ρgh is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since P = F/A, its units are N/m2. If we multiply these by m/m, we obtain N · m/m3 = J/m3, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction. Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and PEg per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity. The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning. Bernoulli's Equation for Static Fluids Let us first consider the very simple situation where the fluid is static—that is, v1 = v2 = 0. Bernoulli's equation in that case is P1 + ρgh1 = P2 + ρgh2. We can further simplify the equation by taking h2 = 0 (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get [latex]{P}_{2}={P}_{1}+\rho {gh}_{1}\\[/latex] This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h1, and consequently, P2 is greater than P1 by an amount ρgh1. In the very simplest case, P1 is zero at the top of the fluid, and we get the familiar relationship P = ρgh. (Recall that P = ρgh and [latex]\Delta{\text{PE}}_{\text{g}}= mgh\\[/latex]) Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is ρgh. Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter. Bernoulli's Principle—Bernoulli's Equation at Constant Depth Another important situation is one in which the fluid moves but its depth is constant—that is, h1 = h2. Under that condition, Bernoulli's equation becomes [latex]{P}_{1}+\frac{1}{2}{{\rho v}_{1}}^{2}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex]. Situations in which fluid flows at a constant depth are so important that this equation is often called Bernoulli's principle. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v2 is greater than v1 in the equation, then P2 must be less than P1 for the equality to hold. In Example 1 from Flow Rate and Its Relation to Velocity, we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is 1.0 × 105 N/m2 (atmospheric, as it must be) and assuming level, frictionless flow. Strategy Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P1. Solution Solving Bernoulli's principle for P1 yields [latex]{P}_{1}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}-\frac{1}{2}{{\rho v}_{1}}^{2}={P}_{2}+\frac{1}{2}\rho \left({{v}_{2}}^{2}-{{v}_{1}}^{2}\right)\\[/latex] Substituting known values, [latex]\begin{array}{c}{P}_{1} = 7 1.01\times 10^{5} \text{ N/m}^{2} +\frac{1}{2}\left(10^{3}\text{ kg/m}^{3}\right)\left[\left(25.5 \text{ m/s}\right)^{2}-\left(1.96 \text{ m/s}\right)^{2}\right]\\ = 4.24\times {10}^{5}\text{ N/m}^{2}\end{array}\\[/latex] Discussion This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure P2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions. Applications of Bernoulli's Principle There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle. People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called entrainment. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in Figure 2. Figure 2. Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling. The airplane wing is a beautiful example of Bernoulli's principle in action. Figure 2(a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See Figure 2(b).) The pressure on the front side of the sail, Pfront, is lower than the pressure on the back of the sail, Pback. This results in a forward force and even allows you to sail into the wind. For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens? Figure 3 shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in Figure 3(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity (v1=0) in front of it, while fluid passing the other tube has velocity v2. This means that Bernoulli's principle as stated in [latex]{P}_{1}+\frac{1}{2}{{\rho v}_{1}}^{2}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex] becomes [latex]{P}_{1}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex]. Figure 3. (a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust. Thus pressure P2 over the second opening is reduced by [latex]\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex], and so the fluid in the manometer rises by h on the side connected to the second opening, where [latex]h\propto \frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex] (Recall that the symbol ∝ means "proportional to.") Solving for v2, we see that [latex]{v}_{2}\propto \sqrt{h}\\[/latex]. Figure 4(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft. Figure 4. Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is [latex]\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex] , and so [latex]h[/latex] is proportional to [latex]\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex] . (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube. Section Summary Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid: [latex]{P}_{1}+\frac{1}{2}{{\rho v}_{1}}^{2}+\rho {gh}_{1}={P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}+\rho {gh}_{2}\\[/latex]. Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding [latex]{P}_{1}+\frac{1}{2}{{\rho v}_{1}}^{2}=

{P}_{2}+\frac{1}{2}{{\rho v}_{2}}^{2}\\[/latex]. Bernoulli’s principle has many applications, including entrainment, wings and sails, and velocity measurement. 1. You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works. 2. Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case. 3. Refer to Figure 1. Answer the following two questions. Why is Po less than atmospheric? Why is Po greater than Pi ? Figure 1. An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v2 is greater than v1), causing the pressure between them to drop (Pi is less than Po). Greater pressure on the outside pushes the car and truck together. 4. Give an example of entrainment not mentioned in the text. 5. Many entrainment devices have a constriction, called a Venturi, such as shown in Figure 5. How does this bolster entrainment? Figure 5. A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators. 6. Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli’s principle. 7. Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer. 8. Why is it preferable for airplanes to take off into the wind rather than with the wind? 9. Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli’s principle to explain these phenomena. 10. Why does a sailboat need a keel? 11. It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train. 12. Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure. 13. A perfume bottle or atomizer sprays a fluid that is in the bottle. (Figure 6.) How does the fluid rise up in the vertical tube in the bottle? Figure 6. Atomizer: perfume bottle with tube to carry perfume up through the bottle. (credit: Antonia Foy, Flickr) 14. If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen? 1. Verify that pressure has units of energy per unit volume. 2. Suppose you have a wind speed gauge like the pitot tube shown in Example 2 from Flow Rate and Its Relation to Velocity. By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer? 3. If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude? 4. Calculate the maximum height to which water could be squirted with the hose in Example 2 from Flow Rate and Its Relation to Velocity if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate. 5. Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of 220 m2? Typical air density in Boulder is 1.14 kg/m3, and the corresponding atmospheric pressure is 8.89 × 104 N/m2. (Bernoulli’s principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.) 6. (a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be 1.29 kg/m3. (The calculation, based on Bernoulli’s principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat. 7. (a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.) 8. (a) Using Bernoulli’s equation, show that the measured fluid speed v for a pitot tube, like the one in Figure 4(b), is given by [latex]v={\left(\frac{2\rho′gh}{\rho }\right)}^{1/2}\\[/latex], where h is the height of the manometer fluid, [latex]\rho′\\[/latex] is the density of the manometer fluid, [latex]\rho\\[/latex] is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h, as stated in the text.) (b) Calculate v for moving air if a mercury manometer’s h is 0.200 m. Bernoulli’s equation: the equation resulting from applying conservation of energy to an incompressible frictionless fluid: P + 1/2pv2 + pgh = constant , through the fluid Bernoulli’s principle: Bernoulli’s equation applied at constant depth: P1 + 1/2pv12 = P2 + 1/2pv22 1. [latex]\begin{array}{c} {P}&=&\frac{\text{Force}}{\text{Area}}, \\ (P)_{\text{units}}&=&\text{N/m}^{2}=\text{N}\cdot\text{m/m}^{3}=\text{J/m}^{3}\\ &=& \text{energy/volume}\end{array}\\[/latex] 3. 184 mm Hg 5. 2.54 × 105 N 7. (a) 1.58 × 106 N/m2 (b) 163 m Bernoulli’s equation is a simple but incredibly important equation in physics and engineering that can help us understand a lot about the behavior of fluids. It describes the relationship between the pressure, velocity and elevation of a flowing fluid.You can watch the video below for an animated introduction to Bernoulli’s equation, or just keep reading to learn more. equation is essentially a statement of the conservation of energy. It tells us that the static pressure (how much the fluid is pressurised), the dynamic pressure (how fast the fluid is moving), and the hydrostatic pressure (how high up the fluid is) of the fluid remain constant along a streamline.Here is the equation, which was first published by Daniel Bernoulli in 1738.The equation reflects the idea that energy is conserved along a streamline because the three terms can be thought of a representing the pressure energy, kinetic energy and potential energy of the fluid. Bernoulli’s equation states that the overall sum of these energies doesn’t change along a streamline – the energy of the fluid is just transferring between these different forms. If the elevation of the fluid increases, for example, the pressure or the velocity must reduce in proportion.In the real world energy isn’t completely conserved – some energy will always dissipate as a fluid flows, because of the viscosity of the fluid. Energy conservation is one of several assumptions that is built into Bernoulli’s equaton. We’ll cover some of the other assumptions later on.What is a streamline?Bernoulli’s equation states that the static, dynamic and hydrostatic pressures are constant along a streamline. But what’s a streamline?In steady flow (flow that doesn’t vary with time), a streamline is defined as the path traced by a single particle within the fluid.Bernoulli’s equation is usually applied to compare the flow conditions between two points on the same streamline, in which case it is written like this:$$P_1 + \frac{\rho_1}{2}{V_1}^2 + \rho_1 g h_1 = P_2 + \frac{\rho_2}{2}{V_2}^2 + \rho_2 g h_2$$Bernoulli’s equation helps us understand how the pressure, velocity and elevation of a fluid are linked. We can use it to calculate the velocity of water exiting a hose, or to figure out if there is a risk of cavitation (low pressure) in a piping system, for example. It does make some simplifying assumptions, but it is a very powerful tool.The Efficient Engineer Summary SheetsThe Efficient Engineer summary sheets are designed to present all of the key information you need to know about a particular topic on a single page. It doesn’t get more efficient than that!Get The Summary Sheets!Let’s look at an example where we apply Bernoulli’s equation to flow through a restriction in a water hosepipe. The objective is to calculate the change in pressure between Point 1 and Point 2, which are on the same streamline.$$P_1 + \frac{\rho_1}{2}{V_1}^2 + \rho_1 g h_1 = P_2 + \frac{\rho_2}{2}{V_2}^2 + \rho_2 g h_2$$In this example we’ll apply Bernoulli’s equation to calculate the change in pressure at the restriction in this hosepipeThe difference in elevation between the two points is negligible ($h_1 = h_2$), so we can remove the elevation terms from Bernoulli’s equation, and re-arrange obtain this equation for the difference in pressure between the two points.$$P_1 – P_2 = \frac{\rho}{2}({{V_2}^2 – V_1}^2)$$The fluid is incompressible, so the mass flow rate between points 1 and 2 must be the same:$$\dot{m}_1 = \dot{m}_2$$$$\rho A_1 V_1 = \rho A_2 V_2$$By re-arranging we can see that the velocity at point 2 is equal to the velocity at point 1 multiplied by the ratio of the areas of the pipe at the two points.$$V_2 = \frac{A_1}{A_2}V_1$$Substituting this equation into the pressure difference equation gives us an equation for the pressure change based only on V_1, A_1, A_2 and the fluid density ρ.$$P_1 – P_2 = \frac{\rho}{2}{V_1}^2(\frac{{A_1}^2}{{A_2}^2}-1)$$Because of the restriction $A_2 < A_1$, which means that the velocity increases between Point 1 and Point 2 but the pressure decreases.The Venturi meter is a flow measurement device that works on this principle – it measures the pressure drop across a converging section of pipe ($P1 – P_2$), and applies Bernoulli’s equation to calculate the fluid flow rate.A Venturi meter measures the pressure drop across a converging section of pipe and applies Bernoulli’s equation to calculate the fluid flow rateAs we saw in the example above, Bernoulli’s equation tells us that if you restrict the flow of water through a hosepipe, the velocity of the water will increase as it passes through the restricton, but the pressure of the water in the restricted section will drop.This might seem counter-intuitive – it feels natural to assume that an increase in velocity results in an increase in pressure, but it makes sense when you think about the conservation of energy. The energy needed to increase the velocity of the fluid comes at the expense of the pressure energy.This is an interesting observation that we can derive from Bernoulli’s equation – for horizontal flow (i.e. when there is no change in the elevation term) if there is an increase in the velocity of the fluid, there must be a decrease in pressure. This observation is known as Bernoulli’s Principle. For horizontal flow an increase in velocity must be accompanied by a decrease in pressure.Bernoulli’s equation can be stated in three different forms, depending on whether each term has units of energy, pressure or distance. The form of the equation shown at the top of this page was the pressure form.$$\frac{P}{\rho} + \frac{V^2}{2} + gh = \mathrm{constant}$$$$P + \frac{\rho}{2}V^2 + \rho gh = \mathrm{constant}$$$$\frac{P}{\rho g} + \frac{V^2}{2g} + h = \mathrm{constant}$$These forms of Bernoulli’s equation are saying the same thing – that energy is conserved along a streamline. They are just using different quantities to express this concept. Which form of the equation you should use depends on what it is you are trying to calculate (energy, pressure or head).Bernoulli’s equation is based on a few assumptions that it’s important to understand to be able to apply it correctly:Inviscid flow – Bernoulli’s equation assumes that the fluid is inviscid, meaning that it has no viscosity. If the fluid is viscous there would be some dissipation of the fluid’s internal energy as it flows, and the idea that energy is conserved along a streamline would not apply. Of course no fluid is truly inviscid, so some judgement needs to be used when deciding if the equation is applicable.Laminar and steady flow – Bernoulli’s equation can only be used if the flow is steady (i.e. it doesn’t vary with time), and it is laminar. The equation is not applicable for turbulent flow. We’ve got a page covering laminar and turbulent flow if you’d like to learn more about these two flow regimes.Incompressible flow – Bernoulli’s equation assumes that the fluid is incompressible. This assumption is usually valid for liquids, but might not be for gases, particularly at high velocities. Laminar and turbulent flow are two fundamentally different flow regimes that can be observed in a flowing fluid.Learn more Viscosity is a measure of the resistance of a fluid to flow, and is defined as the ratio of shear stress to the rate of shear strain in a fluid,Learn more